SOME CALCULATION METHODS FOR DETERMINATION OF THE TRAVEL TIME OF GROUNDWATER

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ABSTRACT

Equations were derived for determination of the travel time of grounwater for some hydrologic conditions: Horizontal and radial-symmetric flow in a confined aquifer to a well in the centre of a circular island; Flow to a fully penetrating and to a partially penetrating channel through uniform recharge from the surface; Radial flow through precipitation on a circular island; Horizontal and radial-symmetric flow to a well in an infinite leaky aquifer with a constant phreatic head; Horizontal and parallel flow in an infinite leaky aquifer with a straight recharge boundary.

INTRODUCTION

Recently interest in the residence time of groundwater is rising due to the increasing risk for groundwater pollution. Especially information is wanted about the time required for polluted groundwater to reach an area of discharge like a river or pumping plant. For instance in catchments for drinking water supply, the travel time of a water particle between a given location of recharge and the extraction point will determine the required protection measures against pollution in different zones around the pumping plant.

In this paper several analytical calculation methods for the determination of the residence time of groundwater for different hydrologic conditions are represented. The equations for the hydraulic head distribution are known from the literature. Although the travel time of groundwater can easily be derived from these relations, solutions of these problems are hardly found in literature.

Direct measurements of flow velocities by tracers are in porous media only applicable for short distance because of the low flow velocities. In inhomogeneous conditions with permeability produced by fractures in consolidated and igneous rocks, analytical solutions will in general fail. Specially in lime stones with wide solution openings and high flow velocities tracers are widely applicable (e.g. Zötl, 1974). Of special interest are those investigations with tritium. Groundwater which originates from precipitation since 1952 shows a high tritium content, due to the testing of nuclear divices. Tritium has been widely used in both tracing and dating of groundwater (e.g. Libby, 1961).

In case of transport of dissolved material a reduction of the concentration gradient occurs analogous to molecular diffusion. This phenomenon is due to molecular diffusion as well as dispersion through the complicated flow pattern of groundwater in the microscopic pore spaces. Moreover processes like solution, precipitation and base-exchange can occur. The following considerations refer to the saturated movement of water of constant density and viscosity through homogeneous aquifers, where transport greatly dominates over diffusion and dispersion.

GENERAL EQUATIONS

The flow velocity in each point of an aquifer can be determined from the hydraulic properties of the subsurface and the hydraulic gradient, with Darcy's law.

$$V = -\frac{K}{n}\frac{dh}{ds} \tag{1}$$

where : V = average flow velocity in the pores of the aquifer;
K = hydraulic conductivity;
n = porosity;
$\frac{dh}{ds}$ = hydraulic gradient.

Replacement of a water particle over a distance ds takes a travel time dt:

$$dt = \frac{ds}{V} \tag{2}$$

The average flow velocity $\bar{V}$ over a distance Δs is given by:

$$V = -\frac{K}{n}\frac{\Delta h}{\Delta s} \tag{3}$$

And the total travel time Δt during replacement Δs is according to (2) and (3):

$$\Delta t = -\frac{n}{K}\frac{(\Delta s)^2}{\Delta h} \tag{4}$$

Most aquifers are thin with respect to their horizontal extent. Thus the groundwater flow lines are generally assumed to be horizontal in an aquifer. In case of a very thick aquifer and for a short distance between the locations of recharge and discharge, this condition is not satisfied and the bending of the flow lines has to be taken into account.

According to Hooghoudt (1940), the maximum depth z_{max} to which a homogeneous and anisotropic aquifer contributes to the discharge is given by the relation:

$$z_{max} = \tfrac{1}{2} L \tag{5}$$

where L is the distance between the divide and the location of discharge. This implies half-circular stream lines. In general, the maximun depth $z(x)_{max}$ of each stream line in such an aquifer can be approximated as half the distance L-x. Where x represents the distance between the divide and the point of recharge. Thus the length of a flow line is given by the relation:

$$s = \tfrac{1}{2} \pi (L-x) \tag{6}$$

The penetration depth of stream lines in an isotropic aquifer z_1 and in an anisotropic aquifer z_2 with identical geometric properties are related as (Vreedenburgh, 1935):

$$z_1 : z_2 = 1 : \sqrt{\frac{K_v}{K_h}} \tag{7}$$

where K_v and K_h are the vertical and horizontal conductivities of the anisotropic aquifer.

Combination of (5) and (7) yields:

$$z(x)_{max} = \frac{1}{2} (L-x)\sqrt{\frac{K_v}{K_h}} \tag{8}$$

In the absence of sufficient data about the hydraulic head pattern or if future situations have to be predicted, the travel time can be determined from the boundary conditions. In the following approximate solutions will be given for several hydrologic conditions with thickness $b<\frac{1}{2}L$.

HORIZONTAL AND RADIAL-SYMMETRIC FLOW IN A CONFINED AQUIFER TO A WELL IN THE CENTRE OF A CIRCULAR ISLAND

Fig. 1 shows the model which implies:

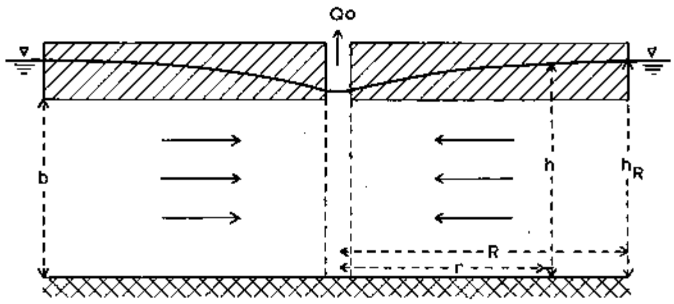

Fig. 1: Horizontal and radial-symmetric flow in a confined aquifer to a well in the centre of a circular isle.

$$Q_o = 2\pi r K b \frac{dh}{dr} \tag{10}$$

where Q_o is the discharge of the well, taken positive, and Kb represents the transmissivity of the aquifer $[L^2 T^{-1}]$.

A combination of (1) and (2) with ds = dr yields:

$$dt = -\frac{n}{K}\frac{dr}{dh}\, dr \tag{11}$$

substitution of (10) into (11) gives:

$$dt = -\frac{2\pi n b r}{Q_o}\, dr \tag{12}$$

Integration with initial condition t = o : r = R leads to:

$$t(r) = \frac{\pi n b}{Q_o}\,(R^2 - r^2) \tag{13}$$

And the total travel time from the boundary to the well:

$$t(o) = \frac{\pi n b R^2}{Q_o} = \frac{\text{total pore volume}}{\text{discharge}} \tag{14}$$

$t_{(o)}^{*}$ represents the time required to exchange the total amount of water in the isle.

The same equations hold for free surface conditions if : $h_R - h \ll h_R$

FLOW TO A CHANNEL IN AN AQUIFER THROUGH UNIFORM RECHARGE FROM THE SURFACE

The flow in a semi-confining layer is assumed to be vertical, and that part of the confining layer which participates in the saturated flow is assumed to have a constant thickness (fig. 2).

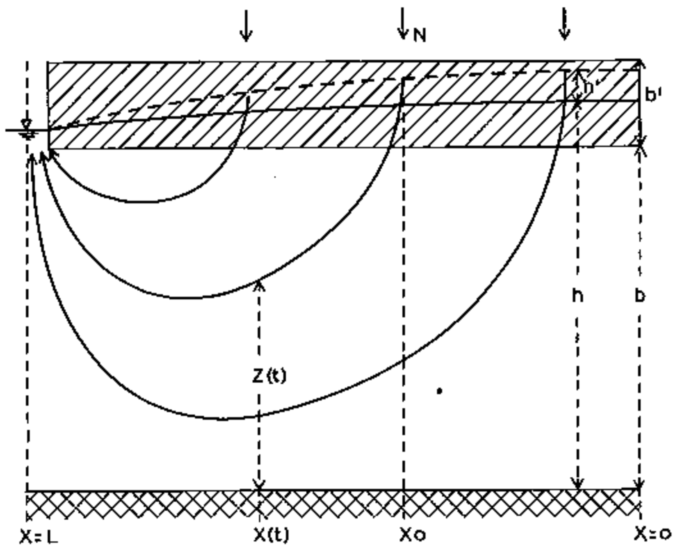

Fig. 2 : Radial flow to a drainage channel

The vertical flow in the semi-pervious layer can analogous to (1) be described by:

$$V' = -\frac{K'}{n'} \frac{dh}{dz} \tag{15}$$

Substitution of (2) with $\Delta s = b'$ and $\Delta h = h'$ yields:

$$t' = \frac{n'}{K'} \frac{b'^2}{h'} \tag{16}$$

where:

- K' = vertical hydraulic conductivity of the confining layer $[LT^{-1}]$;
- n' = porosity (dimensionless);
- b' = thickness of saturated part of the confining layer [L];
- h' = head difference between the phreatic surface and the piezometric surface [L];
- t' = travel time through the confining layer [T]

For uniform recharge N, continuity gives: $n'V' = -N$ (17)

where N represents the recharge $[LT^{-1}]$.

Combination of (15), (16) and (17) yields:

$$V' = -\frac{b'}{t_1} \tag{18}$$

thus $$t' = \frac{n'b'}{N} = \frac{\text{total volume}}{\text{discharge}} \tag{19}$$

If the thickness of the aquifer is small with respect to the horizontal extent of the flow system, the groundwater flow can be approximated as horizontal. This type of flow can be described by:

$$q = -Kb\,\frac{dh}{dx} \tag{20}$$

And:

$$q = Nx \tag{21}$$

where q is the groundwater discharge per unit width.

A combination of (20) and (21) gives:

$$\frac{dh}{dx} = -\frac{Nx}{Kb} \tag{22}$$

From (1) and (2) with ds = dx :

$$dt = -\frac{n}{K}\,\frac{dx}{dh}\,dx \tag{23}$$

Substitution of (22) in (23) yields:

$$dt = \frac{nb}{Nx}\,dx \tag{24}$$

Integrating with initial condition $t = 0 : x = x_o$ gives:

$$t(x) = \frac{nb}{N}\,\ln\,\frac{x}{x_o} \tag{25}$$

where:

- x_o = distance from the divide to the point of infiltration of a water particle;
- x = horizontal distance between the divide and the water particle considered;
- t(x) = travel time as a function of x.

It is evident that in a vertical section the residence time increases with depth, so that in the case of a symmetrical flow pattern for the range x_o to $x = \frac{1}{2}(L + x_o)$:

$$\frac{x(t)}{xo} = \frac{b}{z(t)} \tag{26}$$

where z(t) represents the distance from the impervious base to the water particle at time t.

Substitution of (24) gives the travel time as function of z:

$$t(z) = \frac{nb}{N} \ln \frac{b}{z(t)} \quad (27)$$

Eqs. (25) and (26) yield the travel time of the horizontal flow component. To obtain the total travel time in the aquifer, the vertical flow component has to be taken into account. Analogous to (19):

$$t(z)' = \frac{n(b - z(t))}{N} \quad (28)$$

Substitution of (26) yields:

$$t(z)' = \frac{nb}{N} \left(1 - \frac{xo}{x(t)}\right) \quad (29)$$

The equations hold also for free-surface conditions if the difference in hydraulic head between location x_o and x(t) is small with respect to b. In that case t' = 0.

INFLUENCE OF THE RADIAL FLOW NEAR A DRAINAGE CHANNEL

The consideration in this paragraph have been derived from an unpublished report by Ernst (1973); The solutions in the foregoing section refer to horizontal flow. Therefore they do not hold for the area between x = L and x =L–b, where radial flow predominates through the upward bending of the flow lines.

Analogous to (13) the radial flow can be approximated by (fig. 1):

$$t(r_o) - t(r) = \frac{\pi n}{NL} (r^2 - r_o^2) \quad (30)$$

where Q_o/b of (13) has been replaced by NL.

Transformation of r = L–x yields:

$$t(x_o) - t(x) = \frac{\pi n}{NL} [(L-x)^2 - (L-x_o)^2] \quad (31)$$

If $(L-x_o) \ll (L-x)$, then the second term in (30) can be neglected and for $L-b<x<L$, (30) becomes:

$$t(L) - t(x) = \frac{\pi nL}{4N} \left(1 - \frac{X}{L}\right)^2 \quad (32)$$

The influence of the radial flow can be neglected for small value of b/L; that means in general for $b/L<0.1$.

RADIAL FLOW THROUGH PRECIPITATION ON A CIRCULAR ISLAND

The flow boundary is formed by a drainage channel: fig. 1 shows a radial section of this case, with x = r and L = R.

Analogous to (19): $t_1 = \frac{n'b'}{N}$

In this case (20) becomes:

$$Q = 2\pi r\, Kb\frac{dh}{dr} \tag{33}$$

and (21):

$$Q = \pi r^2 N \tag{34}$$

where Q is now the discharge from a circular area $[L^3T^{-1}]$.

Combination of (33) and (34) yields:

$$\frac{dh}{dr} = -\frac{Nr}{2Kb} \tag{35}$$

Analogous to (25):

$$t(r) = \frac{2nb}{N} \ \ln \frac{r}{ro} \tag{36}$$

From (27):

$$t(r) = \frac{2nb}{N} \ \ln \frac{b}{z(t)} \tag{37}$$

And analogous to (29):

$$t(2)' = \frac{2nb}{N} \ (1 - \frac{ro}{r(t)}) \tag{38}$$

The same equations are valid for an unconfined aquifer if the difference between the hydraulic head between r_o and r(t) is small with respect to b. In that case t' = 0.

HORIZONTAL AND RADIAL-SYMMETRIC FLOW TO A WELL IN AN INFINITE LEAKY AQUIFER WITH A CONSTANT PHREATIC HEAD

The model is represented in fig. 3. Assuming vertical flow in the confining layer, the travel time for groundwater through this layer is analogous to (16):

$$t(r)' = \frac{n'b'^2}{K'(h_c - h)} \tag{39}$$

where h_c is the constant phreatic head in or above the confing layer.

If the radius of the well is negligible and $h_c - h = 0$ if $r \rightarrow \infty$, then the head distribution in the aquifer satisfies the equation (De Glee, 1930):

$$h = h_c - \frac{Q_o}{2\pi Kb} \ K_o\left(\frac{r}{B}\right) \tag{40}$$

where: K_o = Modified Bessel function of first kind and zero order;
B = $\sqrt{Kb\Lambda}$ (leakage factor) [L];
Λ = $\frac{b'}{K'}$ vertical flow resistance of the semi-pervious confining layer [T].

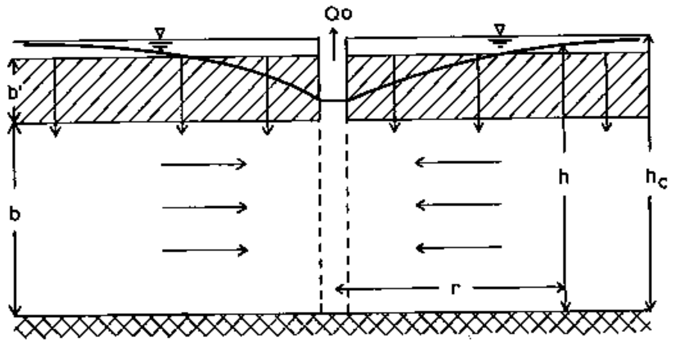

Fig. 3 Horizontal and radial-symmetric flow to a well in an infinite leaky aquifer.

Substitution of (39) in (40) yields:

$$t(r)' = \frac{2\pi n' b'^2 Kb}{Q_o K' K_o(r/B)} = \frac{2\pi n' b' B^2}{Q_o K_o(r/B)} \qquad (41)$$

Replacement of a water particle in the aquifer from a distance r to the well requires a time t(r). From (40):

$$\frac{dh}{dr} = -\frac{Q_o}{2\pi KbB} \; K_1\left(\frac{r}{B}\right) \qquad (42)$$

Integration of (11) with (42) yields:

$$t(r) = \frac{2\pi nb}{Q_o} \int_o^r \frac{dr}{BK_1(r/B)} \qquad (43)$$

or:

$$t(r) = \frac{2\pi nbB^2}{Q_o} \int_o^{r/B} \frac{1}{K_1(x)} \; dx \qquad (44)$$

The function $\int_o^{r/B} \frac{dx}{K_1(x)}$ is given in table 1.

HORIZONTAL AND RADIAL-SYMMETRIC FLOW TO A WELL IN AN INFINITE AQUIFER WITH RECHARGE FROM A PARALLEL SYSTEM OF STREAMS

The water level in the channels is assumed constant; the model is shown in fig. 4.

Analogous to (40) Ernst (1971) obtained the following solution:

$$h = h_c - \frac{Q_o}{2\pi kb} \quad K_o\left(\frac{r}{B}\right)_* \qquad (45)$$

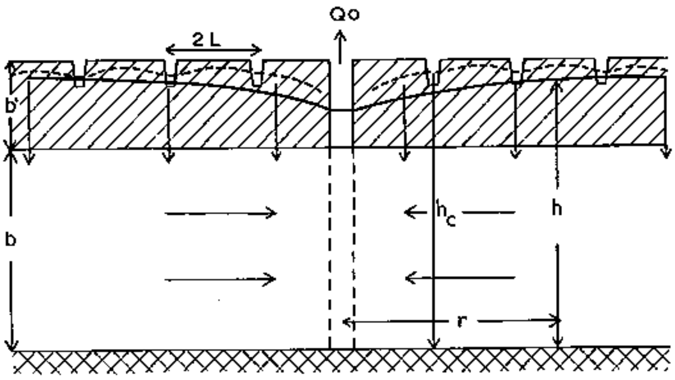

Fig. 4 Radial-symmetric flow to a well in an infinite leaky aquifer with recharge from a parallel stream system.

where: $B^{*} = \sqrt{Kb\gamma}$ [L]

and $\gamma = \frac{L^2}{2Kb} + 2L\Omega$ specific drainage resistance [T] (46)

In (46): L = half stream spacing;

Ω = radial flow resistance [TL^{-1}];

$\Omega = \frac{1}{\pi K'} \ln \frac{4b'}{\pi r_o}$

where r_o represents the radius of a half-circular drainage or infiltration channel.

The average flow velocity from the channels through the confining layer is proportional to the head difference between the channels and the aquifer and inversily proportional with the radial and vertical flow resistance:

$$V' = -\frac{h_c - h}{2nL\Omega} \qquad (47)$$

Hence:

$$t(r)' = \frac{2nb'L\Omega}{h_c - h} \qquad (48)$$

Substitution of (45) in (48) gives:

$$t(r)' = \frac{4\pi nb'KbL\Omega}{Q_o K_o\left(\frac{r}{B*}\right)} \qquad (49)$$

Analogous to (44):

$$t(r) = \frac{2\pi n b B^{*2}}{Q_o} \int_o^{r/B^*} \frac{dx}{K_1(x)} \tag{50}$$

The integral function is given in table 1.

r/B	$\int_o^{r/B} \frac{1}{K_1(x)}\, dx$	r/B	$\int_o^{r/B} \frac{1}{K_1(x)}\, dx$
0.001	0.00000	0.6	0.208
0.004	0.00001	0.8	0.399
0.006	0.00002	1.0	0.679
0.008	0.00003	1.2	1.073
0.01	0.00005	1.4	1.611
0.02	0.00020	1.6	2.334
0.04	0.00080	1.8	3.292
0.06	0.00181	2.0	4.548
0.08	0.00322	2.5	9.565
0.10	0.00504	3.0	18.904
0.2	0.021	3.5	35.902
0.4	0.086	4.0	66.358
		5.0	215.03

Table 1. Some values of the integral functions in Eqs. (44) and (50)

The same equations with $\Lambda = 0$ hold for free-surface conditions.

HORIZONTAL AND PARALLEL FLOW IN AN INFINITE LEAKY AQUIFER WITH A STRAIGHT FLOW BOUNDARY

The phreatic surface in or upon the confining layer is assumed to be constant; the flow boundary represents an open water surface (fig. 5). This model is applicable to seepage underneath a dike into a low polder.

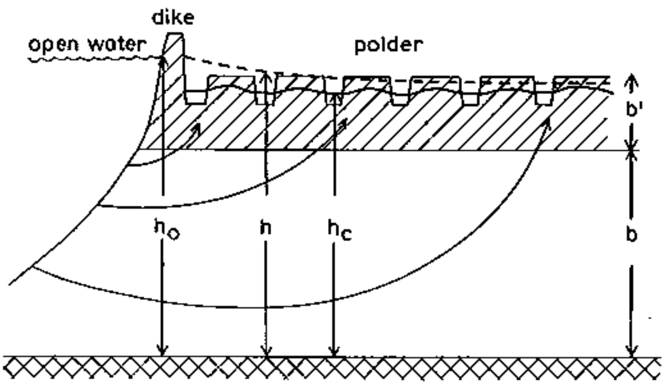

Fig. 5 Diffuse upward seepage in a polder through recharge along a straight boundary.

Flow in the aquifer:

$$q = -Kb\frac{dh}{dx} \tag{51}$$

For continuity:

$$\frac{dq}{dx} = -\frac{h-h_c}{\Lambda} \tag{52}$$

A combination of (51) and (52) gives:

$$Kb\frac{d^2h}{dx^2} - \frac{h-h_c}{\Lambda} = 0 \tag{53}$$

Integration with boundary conditions $h = h_o : x = 0$

$h = h_c : x \rightarrow \infty$

yields:

$$h-h_c = (h_o-h_c)\, e^{-x/B} \tag{54}$$

where: $B = \sqrt{Kb\Lambda}$

Analogous to (16):

$$t(x)' = \frac{n'b'^2}{K'(h-h_c)} \tag{55}$$

Substitution of (54) leads to:

$$t(x)' = \frac{n'b'\Lambda e^{x/B}}{h_o-h_c} \tag{56}$$

From (1) and (2):

$$t(x) = -\frac{n}{K}\frac{dx}{dh}\,dx \tag{57}$$

From (54):

$$\frac{dh}{dx} = -\frac{(h_o-h_c)e^{-x/B}}{B} \tag{58}$$

Substitution of (57) in (58) yields:

$$t(x) = \frac{nBe^{x/B}}{K\,(h_o-h_c)}\,dx \tag{59}$$

Integration with boundaries x_o and x gives the time t(x) that is required to flow from x_o to x :

$$t(x) = \frac{nB^2\,(e^{x/B}-1)}{K\,(h_o-h_c)} \tag{60}$$

or:

$$t(x) = \frac{nb\Lambda(e^{x/B}-1)}{h_o-h_c} \tag{61}$$

EXAMPLES

To give an example of groundwater flow velocities determined by tracers, observations in the dune catchment of the drinking water supply of Amsterdam and observations near Hilversum in an ice-pushed ridge are presented.

In the dune catchment artificial replenishment by water from the River Rhine is induced. River water is infiltrated by shallow infiltration channels and withdrawn as groundwater through a system of drainage tubes parallel to the channels (fig. 6). An increase of the chloride content of the river water is observed in the drainage water with a time lag of about 2 months.

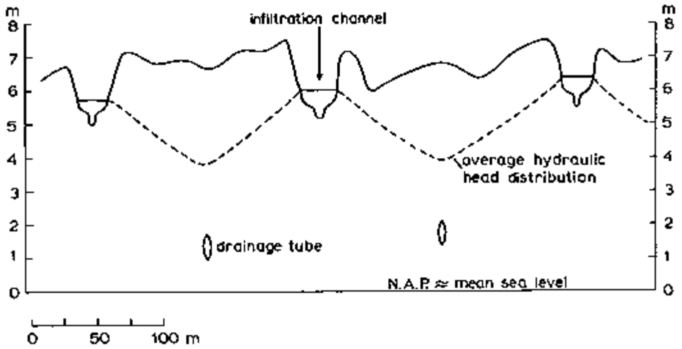

Fig. 6. Part of the infiltration system in the coastal dune catchment of the drinking water supply of Amsterdam

The upper part of the aquifer consists of dune and beach sand with an hydraulic conductivity K of about 10m/day, and a porosity n of 35%. The average distance Δs between the infiltration channels and the drainage tubes amounts 70 m and the average hydraulic gradient 1 : 35. Substitution of these values in equation (4) yields an average travel time T = 85 days. The minimum travel time can be shorter in case of occurrence of layers of coarser material with higher conductivity and smaller porosity.

The influence of the inhomogeneity of the subsurface on the flow distribution in this area has been investigated by Engelen and Roebert (1974) by using the chemical constituents of the infiltrating water as tracers.

Near Hilversum a sewage infiltration plant has been operated for more than 50 years in a high leveled recharge area. The extension of the cloud of polluted groundwater has been traced by its higher chloride content in a great number of observation wells. The average hydraulic gradient shows a value of 1 : 1200. The aquifer consists of coarse fluvial sand with a conductivity of about 35 m/day and a porosity of 35%. The average advance of the front of the polluted water mass appears to be about 25 m/year. This value agrees rather well with equation (1).

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